Decisions, Decisions

By Kevin McKean, Discover Magazine, July 1985.

Few articles have impacted me more than reading this one almost 20 years ago. It has been a mainstay concept shared since I put together the very first IMPACT manual back in 1988 for USA Volleyball. That coaches continued to be "fooled" by their faulty reasoning, is why we continue to teach this concept in all levels of USAV Education. Most recently while roaming the halls of my kids' summer school, I discovered that Kahnema, one of the two scientists covered in this great article, had built upon this, then joined forces with an economist, and had been awarded the 2002 Nobel Prize in Economics. If you want to know more, Tim Gilovich has written a marvelous book on the topic, so important that I gave copies to the National Team coaches back when it came out in 1991. Titled How We Know What Isn't So, it is subtitled on the cover *"The Fallibility of Human Reason in Everyday Life*." ISBN 0-02-911-705-4. I hope you benefit from reading this article we have retyped, as it is so hard to find a copy of it, as much as I have as a parent, coach and teacher. – ***John Kessel***

Threatened by a superior enemy force, the general faces a dilemma. His intelligence officers say his soldiers will be caught in an ambush in which 600 of them will die unless he leads them to safety by one of two available routes. If he takes the first route, 200 soldiers will be saved. If he takes the second, there's a one-third chance that 600 soldiers will be saved and a two-thirds chance that none will be saved.

Which route should he take?

Most people urge the general to take the first route, reasoning that it's better to save the lives that can be saved than to gamble when the odds favor even higher losses. But what about this situation:

The general again has to choose between two escape routes. But this time his aides tell him that if he takes the first, 400 soldiers will die. If he takes the second, there's a one-third chance that no soldiers will die, and a two-thirds chance that 600 soldiers will die. Which route should he take?

In this case, most people urge the general to take the second route.

The first, after all, involves the certain death of 400 men. At least with the second there's a one-third chance that no one with be killed.

And even if the general loses this gamble, his casualties will only be 50 per cent higher.

The fact that most people come to opposite conclusions about these two problems somewhat surprising because, as a cursory inspection reveals, they're identical. The only difference is that the first problem is stated in terms of lives saved, the second in lives lost. When faced with problems like these, people split three to one in favor of the first choice when the question is stated in terms of lives saved, but four to one for the second choice when it's presented as a matter of lives lost. Even when they recognize the contradiction, some people still give conflicting answers.

This paradox is one of many to come out of a remarkable collaboration by two scientists whose work is challenging the fundamental reliability of human reason. The unsettling discovery of Daniel Kahneman and Amos Tversky isn't so much that we're often irrational-everyone knows this. It is that, even when we're trying to be coldly logical, we give radically different answers to the same problem, when it's posed in a slightly different way.

These deviations from rationality, Kahneman and Tversky find, are consistent, and predictable, and result from short cuts the mind takes in weighing complicated odds.

Those familiar with this erroneous reasoning can use it to mold opinion. Moreover, the fact that people's answers flip-flop so readily, even on problems involving life and death, throws into question the very notion that such problems have an answer, ethical, practical, or otherwise. “It means we can't assume our judgments are good building blocks for decisions," says Kahneman, a professor of psychology at the University of British Columbia in Vancouver, "because the judgments themselves may be flawed."

Begun as a simple inquiry into the psychology of uncertainty, Kahneman and Tversky's research has resulted in a theory that provides a systematic explanation for some of the most puzzling aspects of human behavior, and spearheaded the growth of a new discipline of science devoted to the behavioral aspects of decision making. It has won the two scientists an American Psychological Association award and contributed to Tversky's winning a $232,000 MacArthur Foundation fellowship last year. (Friends of both men offer two explanations for why Kahneman didn't also win a MacArthur: First, the fellowships, given to "exceptionally talented" artists and scientists, are never awarded jointly; second, Tversky, a psychology professor at Stanford who was a visiting scholar at Hebrew University in Jerusalem this academic year, has been somewhat more visible outside his field, because of frequent collaborations with scientists from other disciplines.)

Kahneman and Tversky's work has begun to attract the attention of a wider audience-doctors, lawyers, businessmen, and politicians, who see applications for it in choosing therapies, devising legal arguments and corporate strategies, even conducting foreign affairs.

And it has become an object of interest and some controversy-among economists, some of whom see in it a way to factor in the irrational component of human behavior that has confounded so many economic models. "Those who've read the stuff realize that if an answer depends on how a problem is framed, then our whole way of doing business is in trouble," says economist Richard Thaler of Cornell's Graduate School of Management, who's collaborating with Kahneman in applying his and Tversky's research to economics. "There's a widespread feeling among economists that if people's actions aren't logical, then they're random, indescribable. That doesn't leave room for anything between a hyper rational automaton and a blithering idiot, and the people we study are neither."

This recognition didn't come suddenly or easily, but is the fruit of an unusually close 17-year collaboration between two men of such strong and opposing temperaments that mutual friends warned them they would never get along. "Tversky is a sharp and analytical thinker with brilliant mathematical skills. Kahneman, has a more intuitive style; he’s the worrier of the pair" says Baruch Fischhoff, a former graduate student of Kahneman and Tversky's, who's now at a private institute in Eugene, Ore. called Decision Research. "They don't publish a lot compared to some people, but what they do publish is all very polished and very thoughtful."

Kahneman and Tversky papers are dense with mathematical expressed theory, but they've attracted broad attention in part because the authors illustrate each point with a realistic example posed as a question. "We make each example so it can stand on its own," Kahneman says, "and by remembering the example, people remember what it is we were trying to show." The emphasis on practical application may have its origin in the fact that both men are Israelis, and have of necessity developed a strong pragmatic streak. Tversky, 48, whose mother was a member of the first Knesset, is a former paratroop captain who was cited for bravery after saving the life of one of his men during a 1956 border skirmish. Kahneman, 51, set up a psychological screening system in the 1950s that the Israeli army still uses to evaluate recruits. Says Tversky, "Growing up in a country that's fighting for survival, you're perhaps more likely to think about applied and theoretical problems."

It was, in fact, a practical problem that produced one of the original insights that led - by a circuitous route - to Kahneman and Tversky's later work. The incident occurred at Hebrew University in the mid-1960s, when Kahneman, then a junior professor, was teaching a course in the psychology of training to air force Bight instructors. He cited animal studies, some done with pigeons that showed reward to be a more effective teaching tool than punishment. Suddenly one of the flight instructors, barely waiting for Kahneman to finish, blurted out: "With respect. Sir, what you're saying is literally for the birds. I've often praised people warmly for beautifully executed maneuvers, and the next time they almost always do worse. And I’ve screamed at people for badly executed maneuvers, and by and large the next time they improve. Don't tell me that reward works and punishment doesn't. My experience contradicts it." The other flight instructors agreed.

The challenge left Kahneman momentarily speechless. "I suddenly realized," he says, "that this was an example of the statistical principle of regression to the mean, and that nobody else had ever seen this before. I think this was one of the most exciting moments of my career."

Regression to the mean, as Kahneman hastily told the pilots, was the notion worked out by Sir Francis Galton (1822-1911), an English gentleman-scientist, that, in any series of random events clustering around an average or a mean, an extraordinary event was* most likely to be followed, just by luck of the draw, by a rather more ordinary event. Thus very tall fathers tend to have slightly shorter sons, and very short fathers somewhat taller ones. (If they didn't, there would be one-foot-tall and 100-foot-tall people walking around.)

Although regression is usually discussed in narrowly statistical terms, it affects virtually every series of events that is to some degree random. And since there's almost nothing in life that isn't at least partly a matter of chance, regression shows up in a wide variety of unlikely places: it helps explain why brilliant wives tend to have slightly duller husbands; great movies have disappointing sequels, and disastrous presidents have better successors.* The student pilots, Kahneman explained, were improving their skills so slowly that the difference in performance from one maneuver to the next was largely a matter of luck. Regression dictated that a student who made a perfect three-point landing today would make a bumpier one tomorrow-regardless of praise or blame. But the flight instructors, failing to realize this, had underestimated the effectiveness of reward and overestimated the effectiveness of punishment.

Kahneman realized that what had been true of the student pilots would also be true of dancers learning to pirouette, chefs to bake bread, children to obey, and salesmen to close a deal. "By regression alone," he and Tversky later wrote in their book, Judgment Under Cambridge University Press), "behavior is most likely to improve after punishment and to deteriorate after reward. Consequently, the human condition is such that... one is most often rewarded for punishing others and most often punished for rewarding them." The fact that chance plays a role in these matters doesn't rule out other factors, of course. For example, brilliant women may marry duller men to avoid competition, a preference that would magnify a regression-like effect. But even if brilliant women picked only brilliant men, the most brilliant would still have slightly duller husbands, simply because of regression, although the differences between husband and wife would be smaller than if the wives had been less choosy.

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in, the smaller hospital about 15 babies are born each day. Although the overall proportion of boys is about 50 per cent, the actual proportion at either hospital may be greater or less than 50 per cent on any day. At the end of a year, which hospital will have the greater number of days on which more than 60 per cent of the babies born were boys?

a. the large hospital
b. the small hospital
c. neither-the number of days will be about the same (within five per cent of each other)

It wasn't until 1968, when the two men had their first long talk, that Kahneman described the session with the flight instructors to Tversky. Kahneman had invited Tversky, then also a junior professor at the university, to give a guest lecture at a seminar. Afterward, the two struck up a conversation that continued through lunch at the Rimon restaurant, a gathering spot for Jerusalem intellectuals.

"Once you become sensitized to it, you see regression everywhere," Kahneman remembers saying that day. Tversky understood immediately. "Listen to the commentators at the Winter Olympics," he said. "If a ski jumper has done well on his last jump, they say, 'He's under immense pressure, so he's unlikely to do as well this time.' If he did poorly, they say, 'He's very loose and can only improve.' They've detected the effects of regression but invented other causes for it."

The two men decided that it wasn't just that people lacked an intuitive understanding of regression, but that something about regression seemed to run, counter to their intuitions about the ways of the world. Were there other statistical principles that violated common sense?

Tversky could think of one immediately from his study of decision theory. Imagine an urn filled with white balls and black balls. You know that two-thirds of the balls are one color and one-third are the other, but you don't know which color predominates. One blindfolded person plunges a hand into the urn and comes up with three black balls and one white ball. Another uses both hands and comes up with 14 black balls and ten white balls. Both samples suggest that black balls are more numerous. But which sample provides the more convincing evidence?

Many people find the first sample more compelling. After all, it has black in the majority by a three-to-one margin, while the second sample is only a little more than half black. However, probability theory says the odds that the second sample accurately indicates the majority color in the urn are 16 to one, and only four to one for the first sample. The reason is that the first sample is smaller, and small samples are less reliable. Remember that it's fairly common to flip a coin four times and get heads three-quarters of the time. But the chance of obtaining a proportion so out of line with the real odds of 50-50 is exceedingly small after 1,000 flips.

Though people are aware of this fact in an abstract way, Tversky went on, they often ignore it. Squash players think it makes no difference whether they play a game to nine or to 15 points; actually, the shorter game gives the weaker player a much better chance of winning-because he has to win that many fewer lucky points. Similarly, only about one person in five realizes that, in the hospital problem above, the institution with the smaller number of babies per day will have many more days of 60 per cent boys. (The small hospital will have 55 such days a year, on average, the large one 27 days.)

What was going on in people's heads to produce these errors? By trading examples, Kahneman and Tversky came up with several possibilities at the Rimon. "Many of our best ideas were hatched in rudimentary form during that lunch," Tversky says. "It started a conversation that has lasted some seventeen years. We are, in many ways, continuing that conversation today."

THE MIND
Linda is 31, single, outspoken, and very bright. She majored in philosophy in college. As a student, she was deeply concerned with discrimination and other social issues, and participated in antinuclear demonstrations. Which statement is more likely:

a. Linda is a bank teller
b. Linda is a bank teller and active in the feminist movement

Consider, says Tversky, the way mathematics handles uncertainty. "In probability theory, you disassemble an event into all its possible outcomes and then count how many of them have the characteristic you're looking for. Put how do you decide problems such as whether a particular soccer team will win, or whether a patient in therapy is likely to commit suicide, or whether Egyptian president Mubarak is likely to propose a new peace initiative? It isn't clear that even our best analytical methods could handle such problems. So the mind handles them in a very different way. It assesses the evidence intuitively and compares it to some mental model of what a winning team, a suicidal patient, or a peaceful leader is like. If the two match, then the mind concludes that the event is more likely."

This problem-solving method, which Kahneman and Tversky call representative ness, is a short cut the mind takes in dealing with real-world problems that are complicated they would choke a computer. Representativeness works well much of the time, but poorly when its conclusion runs counter to the laws of chance. In the hospital and squash problems, for instance, people wrongly expect the small sample to be as representative of overall probability as the large sample. In the Linda problem, people think she's more likely to be both a bank teller and a feminist, since feminism seems more representative of Linda than being a bank teller. But it's a principle of probability that the likelihood of any two uncertain events happening together is always less than the odds of either happening alone (the chance of flipping two heads in a row is less than the choice of Sipping one). So the odds that Linda is both a teller and a feminist must be less than that she is a teller, regardless of how unsuitable that career may seem for her.

A second mental short cut, on that the scientists call availability, occurs when people judge the likelihood of something happening by how easily they can call other examples of the same thing to mind. Availability, too, appears to be a wonderful way to tackle complex problems because, in general, commoner events are more easily remembered. But consider this: in a typical English text, does the letter K appear more often as the first letter in a word or the third letter? Most people judge that K is commoner at the beginning of words, because it's easier to recall words that begin with a letter than those that contain it somewhere in the middle. Actually K appears about twice as often in the third position.

A more damaging misapplication of availability was investigated by Paul Slovic, Sarah Lichtenstein, and Fischhoff at Decision Research. They found that people overestimate the probability of large, vividly imaginable causes of death (such as airplane accidents, fires, and murder)and underestimate the likelihood of more common but less dramatic causes of death(such as emphysema and stroke), simply because vivid accidents are easier to picture in the mind. Slovic says this helps explain why safety studies of complicated new technologies like nuclear power are so rarely convincing to the public. Engineers try to imagine all the ways a nuclear power plant can fail so they can show how unlikely it is that a large number of small failures could occur together to produce catastrophe. But, says Slovic, "the very act of telling somebody about a low-probability catastrophe and going through the fault tree to show why it's improbable may make the accident more imaginable and memorable, and thus seemingly more likely."

In general, Kahneman and Tversky say, people overestimate the probability of future scenarios that are constructed from a series of individually probable events. "How do people formulate strategy?" Tversky asks. "First they decide what their opponents, say the Russians, are likely to do. Then they plan how they will react. Then they decide how the Russians will react, and so on. It sounds like the right thing to do." The principle of representative ness dictates that the more detailed these future scenarios become, the more likely they will seem-since detail makes an account more strongly resemble the real world. But imagine a scenario involving just seven such assumptions, each of which has a 90 per cent chance of being right. Its overall odds would actually be somewhat less than 50-50 (.9 x .9 x .9 x.9 x .9 x .9 x .9 = 47.8 per cent). Thus, says Tversky, "it may not be wise to choose the course of action that's the best response to what you think the Russians are most likely to do. You may not know what the Russians are up to. Even the Russians may not know what the Russians are up to!"

Not that you should despair.."Even with enormous uncertainties, some courses of action are better than others," says Tversky. "But you shouldn't be very confident of your choices. Actions that acknowledge a high degree of uncertainty are often very different from actions that don't. It's frightening to think that you might not know something, but more frightening to think that, by and large, the world is run by people who have faith that they know exactly what's going on."

A cab was involved in a hit-and-run accident. Two cab companies serve the city: the Green, which operates 85 per cent of the cabs, and the Blue, which operates the remaining 15 percent. A witness identifies the hit-and-run cab as Blue. When the court tests the reliability of the witness under circumstances similar to those on the night of the accident, he correctly identifies the color of a cab 80 per cent of the time and misidentifies it as the other 20 per cent. What's the probability that the cab involved in the accident was Blue, as the witness stated?

The year was 1975, and Kahneman was chairing a planning session for a high school textbook on decision making. No such book had ever been written: in the texts that existed were all for college students. But the committee members felt sure it could be done. "Everybody wrote down an estimate of how long the project would take, and all the estimates were between eighteen and thirty months," Kahneman says. "Then I got the idea of asking one person who had experience in these things how long it had taken other groups to develop curricula for the first time. He looked a little startled and said, 'An interesting thing occurs to me. In the first place, most of them never succeed-they never write a book at all. Of those that do succeed, I can think of hardly any that did it in less than seven years.' "As it turned out, the book, An Elementary Approach to Thinking under Uncertainty (Lawrence Erlbaum Associates), was finished after eight years.

The scientists, all of them skilled in statistics, had fallen into another elementary error: ignoring the so-called base rate-or background data-against which the probability of an event must be judged. The taxi problem is an example of this principle. Most people conclude that if the witness is 80 per cent accurate, then the odds are 80 per cent that the cab was Blue as he stated. In fact, it's more likely the cab was Green. To discover why, imagine that the witness sees 100 hit-and-run accidents instead of just one. By the laws of probability, about 85 of them will involve Green cabs and 15 Blue. Of the 85 Green cabs, the witness will misidentify 20 per cent-or 17 cabs-as Blue. And of the 15 Blue cabs, he'll correctly identify only 80 per cent, or twelve. Thus, of the 29 times the witness says he sees a Blue cab, he's wrong 17 times-an error rate of nearly 60 per cent. The base rate - the preponderance of Green - makes the odds 60 to 40 that he has misidentified a Green cab rather than correctly identified a Blue one.

Consideration of the base rate is another principle that people accept in theory but stubbornly refuse to apply to their own lives. Venture capitalists plunge into start-up companies despite the dismal failure rate of new businesses; newlyweds feel certain their marriage will last despite the appalling rate of divorce; every crook-not to mention every smoker-feels certain that he's the one who'll beat the odds. Says Tversky, "People treat their own cases as if they were unique, rather than part of a huge lottery. You hear this silly argument that 'The odds don't apply to me.' Why should God, or whoever runs that lottery, give you special treatment?"

The issue, says Kahneman, isn't that randomness plays a larger role in life than most of us would like to admit-although that's probably true. But even if the future could be reliably predicted from knowledge of the present, who can be sure which factors in the present are the important ones? "There's a strong overconfidence effect," Kahneman says. "Suppose I take you into a darkened room and show you a circle with no distance cues and ask you how big it is. You don't know whether it's a small circle very close or a large circle far away. If the mind worked like a computer, it would say it doesn't know the answer. But people always have a firm feeling about the circle's size, even when they know they're probably wrong.

"What you see here is a classic example of how the human mind suppresses uncertainty. We're not only convinced that we know more about our politics, our businesses, and our spouses than we really do, but also that what we don't know must be unimportant."

At noon on October 31, 1956, on the third day of the Sinai campaign, a column of Israeli halftracks carrying two companies of paratroopers rumbled into the Mitla Pass, a steep-sided ravine leading to the Suez Canal. Their commanders were taking a calculated risk; Egyptian troops had been seen in the area the day before, but were believed to have been driven off by Israeli air strikes. The column had nearly reached the other end of the ravine when Egyptian soldiers in caves high above opened fire with machine guns and anti-tank weapons. The first two halftracks were hit immediately; a third plunged into a ditch. A fuel truck, an ammunition truck, and three other vehicles went up in flames as the remaining vehicles raced for safety.

Imagine that you face this pair of concurrent decisions. Examine both decisions, and then indicate which choices you prefer:

Decision I.
Choose between:
a. a sure gain of $240
b. a 25 per cent chance of winning $1,000 and a 75 per cent chance of winning nothing

Decision II.
Choose between:
c. a sure loss of $750
d. a 75 per cent chance of losing $1,000 and a 25 percent chance of losing nothing

For the rest of the day and into the night, the paratroopers battled to gain possession of the cliffs and then cleared the caves in hand-to hand fighting. But the toll was heavy: 38 paratroopers killed-nearly a quarter of all Israeli casualties in that brief war. "In officer training school you would be flunked for a decision like that," says Tversky, a member of the paratrooper regiment who was fortunate enough not to be on the ill-fated patrol. "You're supposed to send a small detachment to high ground and make sure no enemy is there, or send a small force through and see if it draws fire.

But in the Israeli army, which has powerful historical motives for taking risks, this kind of decision may be defensible. Look at the rescue at Entebbe. What were the odds there? Maybe fifty-fifty that the operation would be a success, as it turned out to be. Maybe twenty-five per cent that the rescuers and the defending forces would sustain equal casualties. But there was also a possibility-I don't know how high, but not negligible-that it would be a total disaster. If you take a risk like that and win, you're a hero. If you lose ..."

What factors motivate people to gamble or not to gamble? The seventeenth- and eighteenth-century mathematicians who devised probability theory were puzzled by the fact that people seemed reluctant to accept fair risks. For example, few people are willing to pay $500 to enter a lottery with a one-half chance of winning $1,000, although that's the game's mathematically fair price (1/2 x $1,000 = $500).The first satisfactory answer to this problem came from the Swiss mathematician Daniel Bernoulli in 1738. Bernoulli argued that the first dollar-well, actually he used ducats-one acquired was worth slightly more in subjective terms than the second, the second more than the third, and so on, until people with lots of money valued each extra dollar very little.

A consequence of this line of thinking is that to most people the $500 it costs to enter the lottery is worth more than the additional $500 they might win- which explains why most people demand either a lower ante or better odds. This notion that people are "risk averse," as decision theorists put it, has endured into the twentieth century and become part of many economics models. Without risk aversion, it's often said; insurance companies wouldn't be in business. Otherwise, why would people be willing- as millions of us are- to pay insurance premiums that are demonstrably more expensive than the mathematically fair price?

But in the 1960s and '70s, work by Kahneman, Tversky, and a number of colleagues proved that this notion was simplistic. Consider the following example:

Choose between:
a. a sure gain of $3,000
b. an 80 per cent chance of $4,000 and a 20 per cent chance of winning nothing.

In this situation, as expected, most people are averse to risk. They go for the bird-in-hand ($3,000) despite the fact that the alternative has a slightly higher expected pay-off (.8 x $4,000 = $3,200). But when Kahneman and Tversky turned the problem around,

Choose between;
a. a sure loss of $3,000
b. and #0 per cent chance of losing $4,000 and a 20 per cent chance of losing nothing

the preferences are reversed.

More than 90 per cent of the respondents went for the gamble, risking a larger loss for the possibility of losing nothing at all. When Kahneman and Tversky explored further examples, the same dramatic pattern persisted: people tend to avoid risks when seeking gains, but choose risks to avoid sure losses.

The principle, Kahneman and Tversky observe, turns up in many real situations. People need a strong inducement to put money into a gamble. But they expose themselves to tremendous risks in order to avoid a loss, as when a mugging victim resists an armed attacker or when a losing gambler goes for broke. The effect is particularly pronounced in questions of life and death. That's why, in the problems at the beginning this article, people avoid risk when seeking to save lives, but choose it when seeking to avoid deaths.

Overall outcome of A and D:
a 25 per cent chance of winning $240
a 75 per cent chance of losing $760

Overall outcome of B and C:
a 25 per cent chance of winning $250
a 75 per cent chance of losing $750

This example has a cautionary lesson for investment counselors, who must often choose among a variety of more or less risky prospects to put together the best portfolio. Decisions, as it does in the way most people respond to the example on page 28 that presents two concurrent choices. By choosing the sure thing in seeking a gain, but gambling to avoid a loss, three-quarters of the people questioned stumble into choices A and D, which are demonstrate My worse than B and C. The monetary difference between the two pairs of choices isn't large. But when the alternatives are regrouped and each pair of options added together, it's clear that B-C is the superior pair.

You've decided to see a Broadway play and have bought a $40 ticket. As you enter the theater, you realize you've lost your ticket. You can't remember the seat number, so you can't prove to the management that you bought a ticket. Would you spend $40 for a new ticket?

You've reserved a seat for a Broadway play for which the ticket price is $40. As you enter the theater to buy your ticket, you discover you've lost $40 from your pocket. Would you still buy the ticket? (Assume you have enough cash left to do so.)

This propensity would be harmless enough if it didn't sometimes lead to poor paradoxical attitudes toward risk. By this time both men had left Israel, Kahneman for Vancouver and Tversky for Stanford. Yet they continued to collaborate by telephone and letter, and by commuting. "We worked for nearly five years on Prospect Theory to get it right," Kahneman says. "That's characteristic of Amos. He really believes that he can get it right, and he'll stay with it until he does." Adds Tversky, "We slow each other down by an order of magnitude. No sentence gets in unless it is just what we both want. But I'd still rather have Danny's opinion than anybody else's on most issues. We can do things together that we couldn't do alone."

A central tenet of Prospect Theory is hard that even our fastest computers can't factor certain 120- digit decimal numbers in a year," says Blum. This makes the generator very secure.

The fundamental difficulty faced by Blum and others who seek true randomness is that while there are many ways to prove that a sequence isn't random, there's no way to prove that it is. The statistical tests used to evaluate randomness (see boxes) all have an ad hoc quality to them.

A corollary to this problem is the aforementioned lack of agreement on what it means to be random. The definition most often cited is one proposed independently in the 1960's by Soviet mathematician Andrei Nikolaevich Kolomogorov and computer scientist Gregory Chaitin of IBM. They said a sequence is random when the shortest computer generating it must include the sequence itself. That's another way of saying computers can't be compressed.

The Kotmogorov-Chattin definition has the virtue of disqualifying certain numbers like the digit sequence of pie-that pass every known test for randomness yet clearly aren't random. Even so, Kolmogorov-Chaitin provides little help in proving that a particular sequence is random, since it's impossible to know whether there's some simple relationship, analogous to pie, that defines it. Difficulties like this have led some so-called subjectivist statisticians-Diaconis among them-to argue that randomness and other basic notions of probability are as much a matter of mind as of math. "The subjectivist treats probability as representing a person's belief about something not a property of the thing itself Says Diaconis. Confronted with a simple problem that involves chance Diaconis says, people quickly learn from experience to assign probabilities to it. Take the Question of how a coin will fall.

An edge-spinning coin is often very far from "fair"-it may come up 80 per cent heads, for example. "You may not know what to start with," if he says. "But after fifty spins, your estimate and my estimate of the odds of heads are likely to be quite similar.

"Unfortunately, in most important questions involving randomness-such as who will win an election, or whether a nuclear reactor will explode-there's no possibility of careful physical analysis and repeated experiments are impossible. Any discussion of probability must address subjective as much as objective concerns. There's no one right number about the odds of a nuclear disaster," Diaconis says. "People have different opinions, and they should be allowed to have them."

That being the case, it's somewhat dismaying to learn that our intuitions about randomness are often wrong, as shown by the work of psychologists Amos Tversky of Stanford and Daniet Kahneman of Berkeley, among others. The mistake people make, say Tversky and Kahneman, is to expect short runs of random numbers as longruns. "So, in ten flips, you might expect that a coin was random if it came up four heads and six tails, or even three heads and seven tails," says Tversky. "But the moment you see eight or nine heads, you scream bloody murder. Whereas, in fact, that's quite likely to be random."

This misperception leads to two common errors. One is the well known Gambler's Fallacy,in which, for example, people mistakenly, expect red to come up in roulette just because there has been a long run of black. (In fact, if the wheel isn't rigged, the odds of red are the same, regardless of what has gone before.) The other misconception is that, when confronted with the streaks that occur fairly often in any random sequence, people think that they can't be the result of chance. "People see patterns where there are none, "says Tversky, "and they invent causes to explain them."

A dramatic example, from basketball belief in the "hot hand." Athletes and fans alike have long supposed that basketball players shoot in streaks; a player who sinks three or four shots in a row has the hot hand and is more likely to make his next shot. But when Tversky, Robert Vallone of Stanford, and Thomas Gilovich of Cornet) analyzed shooting statistics for four National Basketball Association teams, including the world-champion Boston Celtics, the number of streaks was about what would be expected from chance.

WHEN YOU'RE "HOT,"YOU'RE NOT \

Basketball players and fans commonly believe that players tend to shoot in streaks-that during a game, a player has times when he's hot and every shot goes in, and others when he's cold and barely able to hit the backboard. Sportswriters talk about streak-shooting; players try work the ball to the teammate who has the so-called hot hand, who has made his last three or four shots.

Does the hot hand really exist? To find out, Tversky, and two colleagues Robert Vallone of Stanford and Gilovich of Cornell, interviewed Philadelphia 76ers coach Billy Cunningham and his players about shooting,and then studied detailed records of 48 of the Sixers' games in the 1980-81 season. The players estimated that they were about 25 per cent more likely to make a shot after a hit than after a miss. In fact, the researchers found, the opposite was true-the 76ers were six per cent more likely to score after a miss than after a hit. Darryl Dawkins, for whom this effect was largest, made 71 per cent of the shots he took after misses and 57 per cent of those after hits.

Overall, the number of hot or cold streaks for the 76ers and three other National Basketball Association teams-the Boston Celtics, the New Jersey Nets, and the New York Knicks-was about what would be expected to occur by chance. "There are plenty of excellent reasons why the hot hand could exist," says Tversky. Then why is belief in it so widespread. Tversky says it's because people forget that random sequences, often contain streaks of one sort or another player's try to other; simply by the laws of probability.

For example, there's only a one-in-16 chance of flipping a coin four times and coming up with heads every time. But there's almost a 50-50 chance of getting four heads in a row on any series of 20 flips, a 25 percent chance of five in a row, and a ten percent chance of a streak of six. Since the average NBA player shoots around 50 per cent from the field, he has reasonable odds of making streaks four, five, even six shots in a row if he takes-as an offensive star often would-20 shots in a game. His apparent hot hand will actually just be due to the laws of chance.

Paradoxically, the fact that the players shot better after misses could result from their belief in the hot hand myth. A shooter, who thought he was hot, might take riskier shots; or the opposition, thinking him hot, might guard him closer."This shows that long exposure to chance processes doesn't guarantee that you'll recognize them as such," says Tversky. "In fact, the more basketball you watch, the more you're likely to believe in the hot hand.

Ask Tversky about the hot hand study, and he rolls his eyes>"I've been in an end less number of arguments since then," he says. 'I've won them all, and negotiations underscore their gain-loss aspect," says Tversky. "But because losses loom larger than gains, the other side would have to take two of its missiles out of East Germany before I'd feel justified in taking one of my missiles out of West Germany, and the Russians would react the same way." The best hope for successful negotiations, Kahneman adds, would be if both sides treated their missiles solely as bargaining chips -an attitude that would keep the removal of a missile from feeling like a true loss.

One of the curious features of much of the Kahneman-Tversky work is its apparent simplicity. Many people have the feeling, after they understand one of the examples, that they knew that principle all along. But the feeling is deceptive: like optical illusions, the Kahneman-Tversky paradoxes have a way of fooling people again and again, regardless of how often they've seen them. Says Tversky, "In creating these problems, we didn't set out to fool people. All our problems fooled us, too. Whenever you find an error that statistically naive people make, you can find a more sophisticated version of the same problem that will trip the experts. If I say to my grad students, Do tall fathers always have tall sons?' they think of the textbook page on regression. But if I say something like 'Isn't it interesting that, in French history, the best monarchs usually had average prime ministers, and viceversa?' they say, 'Yes, that's fascinating! I wonder why.'"

The two scientists hope that, as people understand these effects, they will get better at making decisions that involve uncertainty. "Most people find solving a problem quantitatively very unsatisfying," Tversky says, and so they'll re-frame and re-frame the problem until they find a qualitative difference that's decisive. For example, a company may say, "This guy is more productive, but that guy is creative. We need creativity, so we'll hire that guy." This is a very poor way to make a choice. People say love is more important than money," he adds with a twinkle in his eye. "I say a lot of love is better than a little, but a lot of money may be better than a marginal difference in love." The best way to control these illusions, he says, is simply to teach people to recognize them: "Probably the people who've been manipulating the public for so long - salesmen, advertising executives, politicians - know as much about framing as we'll ever know. Understanding their methods may provide and antidote for the rest of us."

One of the fringe benefits of knowing something about framing, Kahneman adds, is that you can use it to make difficult decisions in your life more palatable. He tells the story of when he and his wife, psychologist Anne Treisman, moved to Vancouver and needed to buy a houseful of furniture: "One of the ideas Tversky and I had is that a loss seems less painful when it is an increment to a larger loss than when it is considered alone." Thus it is easier to pay $3,400 for an item that one expected to cost $3,300 than it is to pay $100 for an item that one expected to be free, although the monetary loss in both cases is the same. Mindful of this principle, the Kahnemans arranged to buy all their furniture within a week of buying their Vancouver house-thereby, in effect, making the furniture cost seem as if it were merely a small add-on to the much larger price of the house."Danny is very proud of the fact that he hasn't bought any furniture since," says Thaler jokingly. "That's not true," says Kahneman. "We've bought more furniture. But spending for it has never since been as painless."